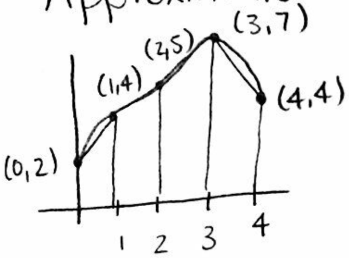
Lesson 33: Numerical Integration (Trapezoidal Rule)

We know FTC, so if we know an antiderivative of $f(x)$ we can find $\int_a^b f(x)dx$. But what if we don't know an antiderivative?

Option 1 use rectangles (R_n, L_n).

Option 2 Use trapezoids to get a better approximation.

Ex 1 Approximate $\int_0^4 f(x)dx$ using 4 trapezoids. $\left(A_{trap} = \frac{1}{2}(b_1+b_2)h\right)$



$$\approx \frac{1}{2}(2+4)(1) + \frac{1}{2}(4+5)(1) + \frac{1}{2}(5+7)(1) + \frac{1}{2}(7+4)(1) \quad \boxed{=19}$$

$$= \frac{1}{2}(1)[2+\underbrace{4+4}_{\downarrow}+\underbrace{5+5}+\underbrace{7+7}+4]$$

$$= \frac{1}{2}\Delta x\,[f(0)+2f(1)+2f(2)+2f(3)+f(4)]$$

Trapezoidal Rule

$$\int_a^b f(x)dx \approx T_n = \frac{1}{2}\Delta x\,[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]$$

T ↑ trapezoid, n ↖ # of trapezoids

$$\underbrace{\Delta x = \frac{b-a}{n}, \quad x_i = a+i\Delta x}_{\text{(same as Riemann sums)}}$$

Ex 2 Approximate $\int_{\overset{\shortparallel}{0}=a}^{4=b} \underbrace{\sqrt{x+1}}_{f(x)}\,dx$ using $\overset{\shortparallel}{8}=n$ trapezoids.

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{8} = \frac{1}{2}, \quad x_0 = 0,\ x_1 = .5,\ x_2 = 1,\ x_3 = 1.5,\ x_4 = 2,\ x_5 = 2.5,$$
$$x_6 = 3,\ x_7 = 3.5,\ x_8 = 4.$$

$$T_8 = \frac{1}{2}\left(\frac{1}{2}\right)\left[\sqrt{0+1} + 2\sqrt{.5+1} + 2\sqrt{1+1} + 2\sqrt{1.5+1} + 2\sqrt{2+1} + 2\sqrt{2.5+1} + 2\sqrt{3+1} + 2\sqrt{3.5+1} + \sqrt{4+1}\right]$$

$$\approx \boxed{6.7812}$$

Note: Actual area is 6.7869, $R_8 = 7.0902$, $L_8 = 6.4721$

Ex 3 Use the table to approximate $\int_0^{40} f(x)dx$:

x	0	10	20	30	40
$f(x)$	12	15	17	13	4

Δx = length of subintervals = 10

$$T_4 = \frac{1}{2}(10)\left[12 + 2(15) + 2(17) + 2(13) + 4\right]$$

$$= \boxed{530}$$